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Surname	Other names

Pearson Edexcel GCE

Centre Number | | | | | |

Candidate Number | | | | |

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 20 June 2017 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
6681/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal*, *x*-*y plane.*]

A bead P of mass 0.08 kg is threaded on a smooth straight horizontal wire which lies along the line with equation $y = 2x - 1$. The unit of length on both axes is the metre. Initially the bead is at rest at the point (a, b). A force $(6\mathbf{i} - 2\mathbf{j})$ N acts on P and moves it along the wire so that P passes through the point (5, 9) with speed 10 m s^{-1}.

Find the value of a and the value of b.

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Question 1 continued

Q1

(Total 7 marks)

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2. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane and* **k** *is a unit vector vertically upwards.*]

A particle of mass 2 kg moves under the action of a constant gravitational force $-19.6\mathbf{k}$ N. The particle is subject to a resistive force $-\mathbf{v}$ newtons, where $\mathbf{v}$ m s^{-1} is the velocity of the particle at time t seconds.

(a) By writing down an equation of motion of the particle, show that $\mathbf{v}$ satisfies the differential equation

$$\frac{d\mathbf{v}}{dt} + 0.5\mathbf{v} = -9.8\mathbf{k}$$

(2)

When $t = 0$, $\mathbf{v} = (4\mathbf{i} - 6\mathbf{j} + 11.6\mathbf{k})$

(b) Find $\mathbf{v}$ when $t = \ln 4$

(8)

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Question 2 continued

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Q2

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3. The position vectors of the points P and Q on a rigid body are $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$m and $(\mathbf{i} - \mathbf{j} + \mathbf{k})$m respectively, relative to a fixed origin O. A force $\mathbf{F}_1$ of magnitude 6 N acts at P in the direction $(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$. A force $\mathbf{F}_2$ of magnitude 14 N acts at Q in the direction $(3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$. When a force $\mathbf{F}_3$ acts at O, which is also a point on the rigid body, the system of three forces is equivalent to a couple of moment $\mathbf{G}$

(a) Find $\mathbf{F}_3$

(5)

(b) Find $\mathbf{G}$

(3)

When an additional force $\mathbf{F}_4 = (\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ N also acts at O, the system of four forces is equivalent to a single force $\mathbf{R}$.

(c) Write down $\mathbf{R}$.

(1)

(d) Find an equation of the line of action of $\mathbf{R}$ in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and t is a parameter.

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Question 3 continued

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Q3

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4. A uniform lamina PQR of mass m is in the shape of an isosceles triangle, with $PQ = PR = 5a$ and $QR = 6a$. The midpoint of QR is T.

(a) Show, using integration, that the moment of inertia of the lamina about an axis which passes through P and is parallel to QR, is $8ma^2$.

(6)

(b) Show, using integration, that the moment of inertia of the lamina about an axis which passes through P and T, is $1.5ma^2$.

[*You may assume without proof that the moment of inertia of a uniform rod, of mass m and length* $2l$*, about an axis perpendicular to the rod through its midpoint is* $\frac{1}{3}ml^2$]

(4)

The lamina is now free to rotate in a vertical plane about a fixed smooth horizontal axis A which passes through P and is perpendicular to the plane of the lamina. The lamina makes small oscillations about its position of stable equilibrium.

(c) By writing down an equation of rotational motion for the lamina as it rotates about A, find the approximate period of these small oscillations.

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Question 4 continued

Q4

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5. A uniform rod AB, of mass M and length $2L$, is free to rotate in a vertical plane about a smooth fixed horizontal axis through A. The rod is hanging vertically at rest, with B below A, when it is struck at its midpoint by a particle of mass $\frac{1}{2}M$. Immediately before this impact, the particle is moving with speed u, in a direction which is horizontal and perpendicular to the axis. The particle is brought to rest by the impact and immediately after the impact the rod moves with angular speed ω.

(a) Show that $\omega = \dfrac{3u}{8L}$

(4)

Immediately after the impact, the magnitude of the vertical component of the force exerted on the rod at A by the axis is $\dfrac{3Mg}{2}$

(b) Find u in terms of L and g.

(4)

(c) Show that the magnitude of the horizontal component of the force exerted on the rod at A by the axis, immediately after the impact, is zero.

(2)

The rod first comes to instantaneous rest after it has turned through an angle α.

(d) Find the size of α.

(5)

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Question 5 continued

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6. A small object P, of mass m_0, is projected vertically upwards from the ground with speed U. As P moves upwards it picks up droplets of moisture from the atmosphere. The droplets are at rest immediately before they are picked up. In a model of the motion, P is modelled as a particle, air resistance is assumed to be negligible and the acceleration due to gravity is assumed to have the constant value of g. When P is at a height x above the ground, the combined mass of P and the moisture is $m_0(1 + kx)$, where k is a constant, and the speed of P is v.

(a) Show that, while P is moving upwards

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(v^2\right) + \frac{2kv^2}{(1 + kx)} = -2g$$

(7)

The general solution of this differential equation is given by $v^2 = \frac{A}{(1 + kx)^2} - \frac{2g}{3k}(1 + kx)$, where A is an arbitrary constant.

Given that $U = \sqrt{2gh}$ and $k = \frac{7}{3h}$

(b) find, in terms of h, the height of P above the ground when P first comes to rest.

(5)

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TOTAL FOR PAPER: 75 MARKS

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